

Mathematics: analysis and approaches**Standard Level****Paper 1**

Name

Date: _____

1 hour 30 minutes

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

exam: 9 pages

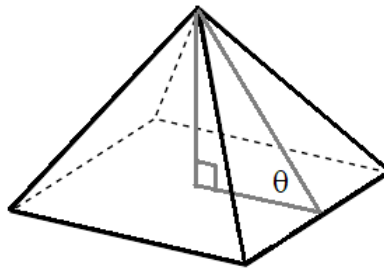
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider the right square pyramid shown below. Given that the area of the square base is 36 cm^2 and the volume of the pyramid is $36\sqrt{3} \text{ cm}^3$, find the angle θ between the base of the pyramid and one of its lateral faces.



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2. [Maximum mark: 5]

Let A and B be events such that $P(A \cap B) = \frac{1}{5}$, $P(B | A) = \frac{1}{2}$ and $P(A | B) = \frac{3}{10}$.

Find $P(A \cup B)$.

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4. [Maximum mark: 6]

Let $h'(x) = x\sqrt{1-x^2}$. Given that $h(0) = \frac{2}{3}$, find $h(x)$.

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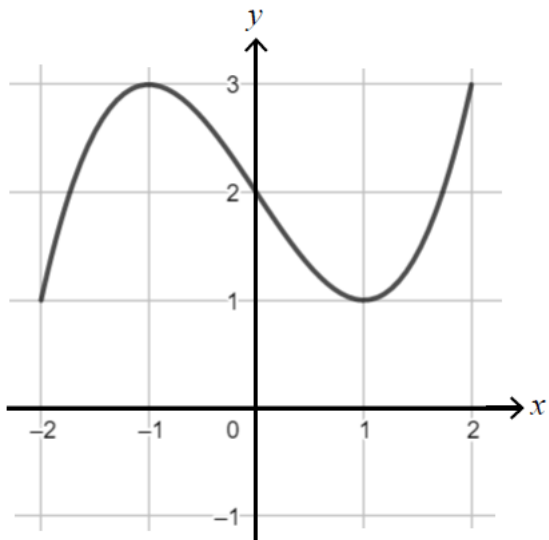
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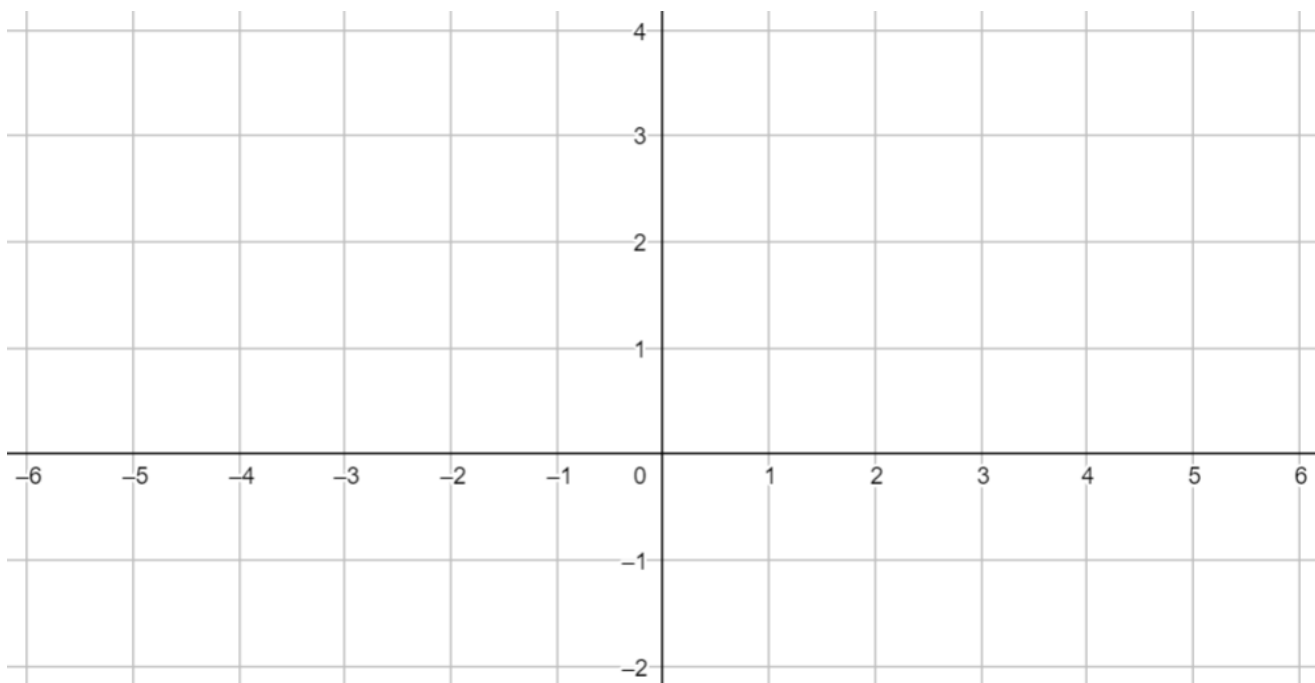
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5. [Maximum mark: 5]

The following diagram shows the graph of $y = f(x)$, $-2 \leq x \leq 2$. The graph has a horizontal tangent at the points $(-1, 3)$ and $(1, 1)$.



On the set of axes below, sketch the graph of $y = f[2(x-2)]$, clearly indicating the coordinates of any local maxima or minima.



6. [Maximum mark: 8]

Solve for x in each equation.

(a) $\ln x + \ln(x - 2) - \ln(x + 4) = 0$ [4]

(b) $\log_3(4x^2 - 5x - 6) = 1 + 2\log_3 x$ [4]

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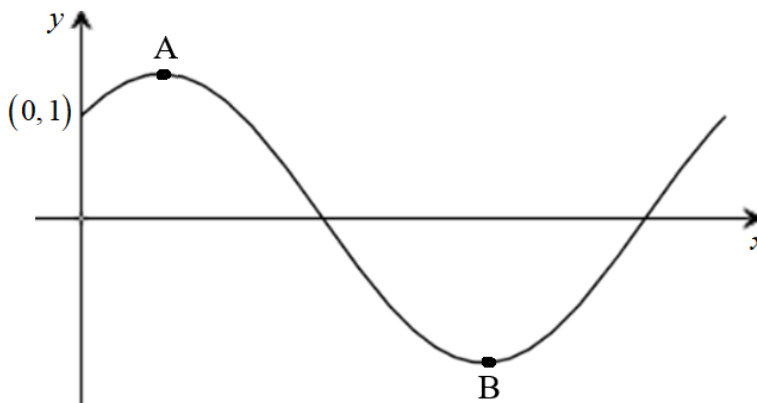
Do **not** write solutions on this page.

Section B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

7. [Maximum mark: 17]

Consider the function f (graph shown below) defined by $f(x) = \cos x + \sin x$, $0 \leq x \leq 2\pi$.



The y -intercept is at $(0, 1)$, there is a maximum point at $A(p, q)$ and a minimum point at B .

(a) Find $f'(x)$. [2]

(b) Hence

(i) show that $q = \sqrt{2}$;

(ii) verify that A is a maximum. [10]

(c) State the coordinates of B . [3]

The function $f(x)$ can be written in the form $r \cos(x - c)$ where $r, c \in \mathbb{R}$.

(d) Write down the value of r and the value of c . [2]

Do **not** write solutions on this page.

8. [Maximum mark: 12]

A bag A contains 2 red balls and 3 yellow balls. A bag B contains 4 red balls and 2 yellow balls. Two balls are randomly selected from one of the bags. If bag A is chosen, then the probabilities are as follows:

$$P(2 \text{ red balls}) = \frac{1}{10}$$

$$P(2 \text{ yellow balls}) = \frac{3}{10}$$

$$P(1 \text{ red ball and 1 yellow ball}) = \frac{6}{10}$$

- (a) Calculate the probabilities for the same three outcomes if bag B is chosen. [5]

In order to decide which bag to choose, a fair die with six faces is rolled. If a 1 or 6 is rolled, then bag A is chosen. If a 2, 3, 4 or 5 is rolled, then bag B is chosen.

The die is rolled and then two balls are drawn from the selected bag.

- (b) Calculate the probability that two red balls are selected. [3]
- (c) Given that two red balls are obtained, find the probability that a 1 or 6 was rolled on the die. [4]

9. [Maximum mark: 18]

The function g is defined by $g(x) = \frac{x}{e^{x^2}}$, where $x \geq 0$.

- (a) Show that there is one maximum point P on the graph of g , and find the x -coordinate of P. [5]
- (b) Show that g has a point of inflexion Q at $x = \sqrt{\frac{3}{2}}$. [6]
- (c) Determine the intervals on the domain of g where g is
- (i) concave up
 - (ii) concave down. [2]
- (d) The region bounded by the graph of g , the x -axis and the vertical line $x = k$ has an area equal to $\frac{1}{2} - \frac{1}{2e^4}$. Find the value of k . [5]